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Calculation of Two-Dimensional Near and Far Wakes

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Several sets of recent experimental data have been used to test the applicability of a two-equation turbulence model in the rapidly relaxing turbulent flow in near wakes and to assess the success with which boundary-layer calculation methods can be used to predict development of near wakes and recover the well-known asymptotic state of far wakes. The results obtained to date suggest that the major characteristics of symmetric as well as asymmetric near wakes can be predicted with satisfactory accuracy but the agreement between prediction and experiment deteriorates with distance from the trailing edge, and the asymptotic growth rates are underestimated.

Nomenclature

b	= half-width of wake
$c_1,c_2,c_\mu,\sigma_k,\sigma_\epsilon$	= constants in turbulence model, Eqs. (13) and (14)
H	= shape parameter ($=\delta^*/\theta$)
k	= turbulent kinetic energy
p	= pressure
s	= growth rate parameter
u,v	= mean-velocity components (Fig. 1)
U	= freestream velocity
w	= velocity defect
w_0	= maximum velocity defect
x,y	= streamwise and normal distances (Fig. 1)
γ	= intermittency
δ	= boundary-layer thickness
δ^*	= displacement thickness
ϵ	= rate of turbulent energy dissipation
θ	= momentum thickness
ν	= kinematic viscosity
ν_T	= eddy viscosity
ρ	= fluid density
τ	= shear stress
ψ	= stream function
ω	= normalized stream function, Eq. (15)

Introduction

THE flow in the near wake of a streamlined two-dimensional body provides a simple and yet a critical test of the generality of turbulence models and calculation procedures since it is a region of relaxation from wall-dominated turbulent boundary layers to a single equilibrium shear layer, the asymptotic or far wake, at large distances from the trailing edge. Almost all turbulence models have evolved from the extensive data bases in boundary layers and fully developed free shear flows such as wakes and jets. The near wake, being the region of adjustment between two extreme states, therefore offers an independent test of the generality of turbulence models. Prediction of the flow in the near wake by the continuation of boundary-layer calculations beyond the trailing edge is also of considerable practical interest in airfoil calculations since the near wake participates in the viscous-inviscid interaction at the trailing edge.

Until recently, the only set of detailed two-dimensional near wake data available was that of Chevray and Kovasznay,[1] who made measurements of mean-velocity and turbulence profiles in the symmetric wake of a thin flat plate. Their data have been employed by numerous workers to test the performance of various turbulence models. Thus, for example, Bradshaw[2] has used a one-equation turbulent kinetic energy model; Burgraff[3] adopted the Cebeci-Smith eddy viscosity model as well as the one-equation model of Glushko; Morel and Torda[4] and Huffman and Ng[5] have proposed special extensions of such models for the treatment of wakes; and Rodi,[6] Launder et al.,[7] and Leuchter[8] have employed several one- and two-equation models. The data of Chevray and Kovasznay were also adopted as a test case at the 1972 NASA Langley Conference on Free Turbulent Shear Flows,[9] at which the results of several prediction methods were compared with the data. Although the comparisons were limited to decay of the centerline velocity defect, it became apparent that the problem of prediction of near wakes was not resolved satisfactorily. Fortunately, availability of several sets of new data has made it possible to re-examine the situation more thoroughly. The purpose of this paper is to present the results obtained with a representative, state-of-the-art two-equation model of turbulence, namely, the k-ϵ model, for several sets of symmetric and asymmetric wake data to assess the performance of the model.

Data Base and Selection of Test Cases

In preparation for the 1980-81 AFOSR-HTTM-Stanford Conferences on Complex Turbulent Flows, Patel[10] reviewed experiments in two-dimensional near wakes and found that the data base has expanded considerably in recent years. Several experiments have been performed since the study of Chevray and Kovasznay,[1] and others are in progress.

The most extensive measurements have been made in the simplest of wake flows, namely, the symmetric wake of a smooth flat plate. These experiments are summarized in Table 1. Patel's review of these data indicated that the older measurements of Chevray and Kovasznay[1] have been superseded by the recent and more extensive data of Andreopoulos,[11] Pot,[12] and Ramaprian et al.[13]

There are several sets of data pertaining to development of symmetric wakes in externally imposed or self-induced longitudinal pressure gradients. The most recent, and perhaps the most detailed, data are due to Viswanath et al.,[14] who made mean-flow as well as turbulence measurements with an airfoil-like model.

Asymmetric near wakes, originating from unequal upper- and lower-surface boundary layers, have been explored for many years, but detailed measurements required for testing calculation methods have been made only recently. The study of Viswanath et al. provides asymmetric wake data. Andreopoulos[11] and Ramaprian et al.[13] have utilized roughness on one side of a flat-plate model to realize the asymmetry. Other experiments in progress in the wakes of airfoils at incidence will provide additional data sets in this category.

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All of the aforementioned measurements relate to near wakes starting from attached flow. The wakes of bluff bodies and airfoils at large incidences have also been investigated experimentally, but measurements in the separated flow are either uncertain or have been avoided altogether.

For the purposes of investigating the performance of turbulence models in near wakes, we have selected the simplest test cases, namely, the symmetric wake of a smooth flat plate and the asymmetric wake of a plate with roughness on one side. In the first category, the data of Andreopoulos,[11] Pot,[12] and Ramaprian et al.[13] have been used since each contains certain unique features. For the asymmetric wake, calculations have been performed for the experiments of Andreopoulos and Ramaprian et al. since the level of asymmetry is different in the two cases. It should be noted that, in the test cases selected, the problems associated with longitudinal pressure gradients, wake curvature, circulation on the body, and reversed flow have been avoided. The mean streamlines are essentially parallel, and the self-induced pressure gradients due to the weak viscous-inviscid interaction at the trailing edge may be neglected in the first instance. Successful solutions of such flows is an obvious first step toward development of methods for the more complex cases. The primary requirement of a calculation method is that it should predict the observed features of near wakes with respect to the mean-flow as well as the turbulence parameters of the model and recover the well-known asymptotic state at large distances from the body.

The Far Wake

At sufficiently large distances from the body, the velocity defect w becomes small compared with velocity u, and the assumption of similarity of velocity and shear-stress profiles,

$$w/w_0 = f(y/b) \qquad \tau/\rho w_0^2 = g(y/b) \tag{1}$$

reduces the equations of motion to an ordinary differential equation that yields the streamwise growth rates of the length and velocity scales of the wake of the form

$$b \sim x^{1/2} \qquad w_0 \sim x^{-1/2} \tag{2}$$

The notation is indicated in Fig. 1.

In order to evaluate the functions in Eq. (1) and the proportionality constants in Eq. (2), it is necessary to invoke a turbulence-closure model. Thus, for example, introduction of an eddy viscosity ν_T that remains constant across the wake leads to (see, for example, Schlichting[15])

$$w/w_0 = \exp[-4\ln 2 (y/b)^2] \equiv f(y/b) \tag{3}$$

$$\tau/\rho w_0^2 = (\nu_T / w_0 b) f' \tag{4}$$

$$w_0 b / U\theta = [(4/\pi)\ln 2]^{1/2} \tag{5}$$

$$(b/\theta)^2 = 16 \ln 2 (\nu_T / U\theta)(x/\theta) \tag{6}$$

$$(U/w_0)^2 = 4\pi (\nu_T / U\theta)(x/\theta) \tag{7}$$

where

$$\theta = \int_{-\infty}^{\infty} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy \tag{8}$$

is the momentum thickness of the wake.

From an examination of some early measurements in the wakes of cylinders, Schlichting deduced $\nu_T/U\theta = 0.0444$. Townsend,[16] however, quotes a value of 0.032, which has been confirmed by Rodi[17] from a survey of several sets of data and by Narasimha and Prabhu[18] in more recent experiments. We shall therefore adopt $\nu_T/U\theta = 0.032$.

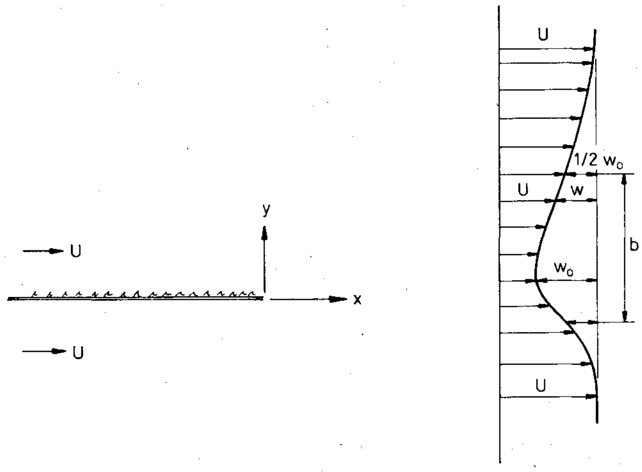

Fig. 1 Notation.

It should be emphasized that the above results are derived on the basis of a constant eddy-viscosity model, which is not entirely realistic in view of the marked intermittency of the flow over a large part of the wake. Nevertheless, they serve as a basis for comparison between data obtained in far wakes originating from different initial conditions and to quantify the term "near wake" used, rather loosely, to refer to the development zone preceding the asymptotic state. We shall use these results also to gage the approach of the solutions to the asymptotic state.

Equations and Solution Procedure

The continuity and momentum equations for wakes are identical with the boundary-layer equations, i.e.,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{9}$$

$$\frac{Du}{Dt} = -\frac{1}{\rho}\frac{dp}{dx} + \frac{1}{\rho}\frac{\partial \tau}{\partial y} \tag{10}$$

where $\tau = \mu(\partial u/\partial y) - \rho\overline{u'v'}$ is the total shear stress. For the wakes to be considered here, $dp/dx = 0$, and the viscous contribution to the stress can be neglected except possibly in a small region close to the trailing edge whose dimensions are of the order of the thickness of the sublayer of the upstream boundary layer. The influence of pressure gradients, if present, can be incorporated quite readily.

Some previous calculations by Ramaprian et al.[13] indicated that a one-equation turbulence model, based on the turbulent kinetic energy equation with a prescribed length-scale distribution, failed to predict the essential features of the near wake and led to a gross underestimation of the asymptotic growth rates. These results were similar to those of one of the models employed by Launder et al.[7] It was also demonstrated that somewhat better predictions were made by the two-equation k-ϵ turbulence model.

Although the long-range objective of the present work is to test the performance of several competing turbulence models against data from different types of near wake flows, in this paper the attention is focused on the k-ϵ model, which has been applied to a wide variety of internal and external shear flows. In this model, the shear stress is related to the mean rate of strain through an eddy viscosity ν_T, i.e.,

$$\frac{\tau}{\rho} = \nu_T \frac{\partial u}{\partial y} \tag{11}$$

where

$$\nu_T = c_\mu (k^2/\epsilon) \tag{12}$$

The turbulent kinetic energy k and its rate of dissipation ϵ are obtained from the model equations

$$\frac{Dk}{Dt} = \nu_T\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\partial}{\partial y}\left(\frac{\nu_T}{\sigma_k}\frac{\partial k}{\partial y}\right) - \epsilon \tag{13}$$

$$\frac{D\epsilon}{Dt} = c_1\frac{\epsilon}{k}\nu_T\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\partial}{\partial y}\left(\frac{\nu_T}{\sigma_\epsilon}\frac{\partial \epsilon}{\partial y}\right) - c_2\frac{\epsilon^2}{k} \tag{14}$$

The five model constants are assigned the following "standard" values:

$$c_\mu = 0.09,\; c_1 = 1.44,\; c_2 = 1.92,\; \sigma_k = 1.0,\; \sigma_\epsilon = 1.3$$

An important objective of the present work was to develop a method for calculation of symmetric as well as asymmetric wakes, since almost all previous methods either have been restricted to the relatively simple symmetric case or have superposed two separate solutions of the hypothetical "shear layers" resulting from the upper and lower surface boundary layers on the body. The starting point for the development of the new method was provided by a computer program available for the calculation of boundary layers.

Equations (9-14) are solved numerically by the implicit finite-difference technique of Patankar and Spalding.[19] A characteristic feature of this method is the use of the normalized stream function

$$\omega = \frac{\psi - \psi_I}{\psi_O - \psi_I} \qquad \psi = \int_{y_I}^{y} \rho u \mathrm{d}y + \psi_I \tag{15}$$

as an independent variable in place of the normal distance y, where subscripts I and O denote the inner and outer edges of the computation domain. This transformation allows calculations to proceed along lines of constant ω, with a constant number of grid points confined to the shear layer by the use of Eq. (15). Also, this feature of the method enables calculations of symmetric as well as asymmetric wakes without special changes for the latter. In both cases, the y coordinate is determined from the transformation by tracking the position of certain ω = const lines relative to the trailing edge, e.g., the trailing-edge streamline in the symmetric case and the two lines at the edges of the wake in the asymmetric case. The calculations for the symmetric case could also be performed by imposing the symmetry boundary conditions, $(\partial/\partial y)(u,k,\epsilon) = 0$, at $y = 0$. In this case, the results by both methods were identical.

Grid-independent solutions were obtained with typically 60 nodes across the wake and streamwise steps of the order of 0.01 wake widths in the near wake, where the normal and streamwise gradients are large. Much larger step sizes could be taken in the far wake, but no attempt has been made to optimize these variables. Computation times were of the order of 0.05 s per forward step on a Burroughs B 7700 computer.

All calculations in the present study were started at the trailing edge, $x = -0$, with the initial conditions provided by experimental data, to the extent these were available, and standard smooth or rough flat-plate boundary-layer characteristics for the remaining quantities. The mean-velocity and shear-stress profiles were available, but in most cases, k and ϵ distributions had to be inferred either from incomplete information or by requiring internal consistency with the turbulence model. In all cases, the external pressure gradient and freestream turbulence were neglected.

Symmetric Wakes

As shown in Table 1, the experiments of Andreopoulos and Ramaprian et al. covered relatively short distances ($x < 80\theta$) from the trailing edge but document the flow relaxation in the near wake in considerable detail. On the other hand, the measurements of Pot extend up to $x \sim 1000\theta$. In the interest of brevity, only the overall parameters will be presented for all three cases. The data of Ramaprian et al. will be used to demonstrate the detailed performance of the calculation method in the near wake and those of Pot to provide a test for the approach of the calculations to the asymptotic state in the far wake.

Figures 2-4 show that the present calculations are in excellent agreement with data in the near wake ($x \sim 350\theta$, say) with respect to the two usual overall parameters, namely, the

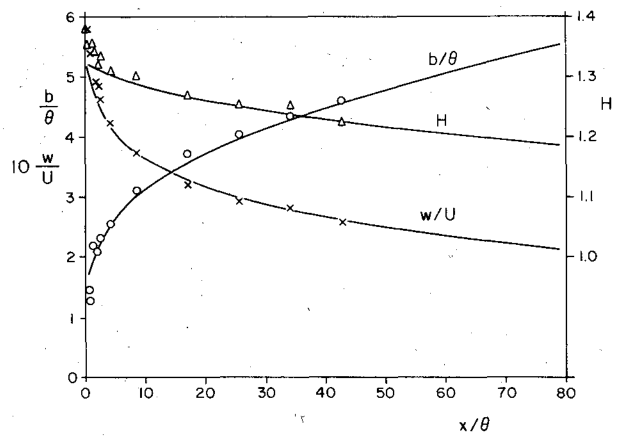

Fig. 2 Symmetric near wake, data of Andreopoulos.[11]

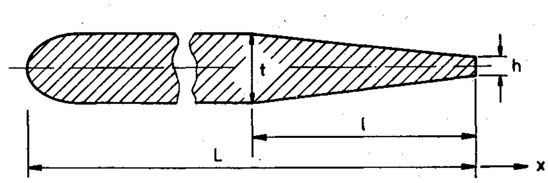

Table 1 Summary of smooth flat-plate wake experiments

Authors	Model				Boundary layer at $x = 0$				Data range
	L, m	t, mm	h, mm	l, mm	$u_\tau h/\nu$	R_θ	H	c_f	x/θ
Chevray and Kovasznay[1]	2.4	1.6	0.25	600	3.1	1,580	1.44	0.0042	0-207
Fayet et al.[20]	0.3	6.0	0.05	35	4.3	3,000	1.37	(0.0028)	0-373
Andreopoulos[11]	3.08	28.0	0.07	450	5.1	13,600	1.39	0.0023	0-43
Pot[12]	0.5	9.5	1.1	—	105	2,940	1.39	0.0031	3-948
Ramaprian et al.[13]	1.80	19.0	1.0	670	58	5,220	1.30	0.0029	10-79

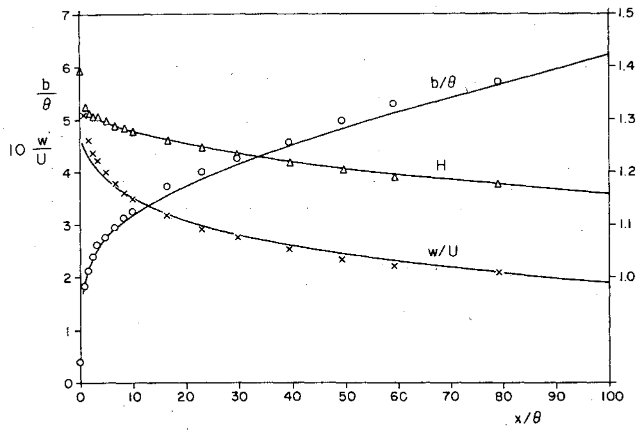

Fig. 3 Symmetric near wake, data of Ramaprian, Patel, and Sastry.[13]

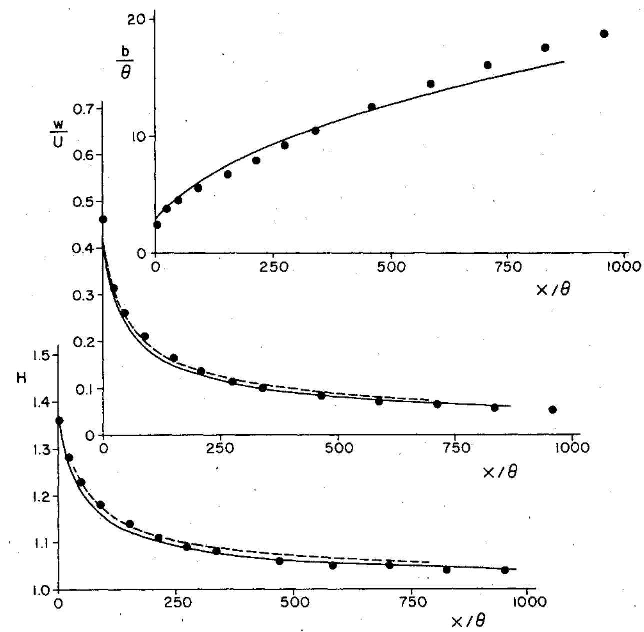

Fig. 4 Symmetric near wake, data of Pot.[12] --- basic model; —— with intermittency.

half-width b and the maximum velocity defect w_0. Since the momentum thickness θ remains essentially constant in these flows, the shape parameter H $(=\delta^*/\theta)$ also indicates development of the displacement thickness δ^*, a quantity of primary concern in the study of viscous-inviscid interaction at the trailing edge. The calculations appear to reproduce the rapid initial decrease in H that occurs within a distance of the order of two momentum thicknesses or roughly 30 upstream sublayer thicknesses. This suggests that neglect of the viscous and normal stress terms in the equations as well as the pressure gradients induced by the viscous-inviscid interactions may be justified for these simple wake flows. The general level of agreement between the calculations and experiments may be considered satisfactory in the near wake, say, $x/\theta<350$, but it is evident from the data of Pot shown in Fig. 4 that there are substantial differences beyond this point. This feature of the calculations in the far wake is discussed later on.

The calculated profiles of mean velocity u, Reynolds shear stress $\tau=-\rho\overline{u'v'}$, and turbulent kinetic energy k are compared with the data of Ramaprian et al. at a few representative stations in Fig. 5. It is seen that all features are predicted quite well except the pronounced local increase in the stress and turbulent kinetic energy near the wake cen-

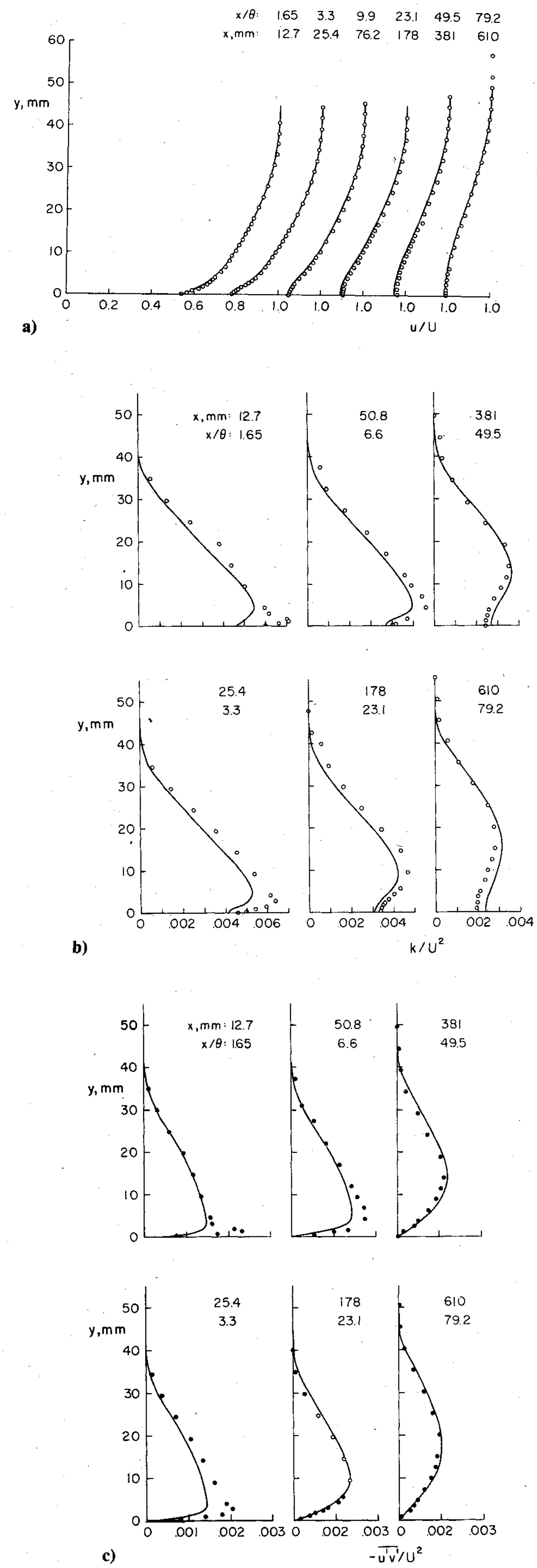

Fig. 5 Symmetric near wake, detailed comparisons with data of Ramaprian et al.[13] a) Mean velocity; b) turbulent kinetic energy; c) Reynolds shear stress.

terline close to the trailing edge. Such high turbulence levels were not recorded in the experiment of Andreopoulos but were observed by Pot. Ramaprian et al. have attributed these to possible vortex shedding or separation due to the relatively thick trailing edges in their and Pot's experiments (see Table 1). Beyond this region, which extends up to $x/\theta \sim 10$, agreement with the data is good up to the most downstream station, $x/\theta = 79.2$, and there is no hint of a subsequent deterioration of the agreement in the far wake.

In order to examine the behavior of the solutions at large distances from the trailing edge, it is convenient to make comparisons on the basis of the asymptotic forms of Eqs. (3-7). Figure 6 shows the streamwise variation of b and w_0 according to the data of Pot. It is important to mention that the best fit of Eqs. (6) and (7) to his data in the range $400 < x/\theta < 1000$ indicates $\nu_T/U\theta \sim 0.035 \pm 0.001$, somewhat higher than the value of 0.032 quoted earlier. Nevertheless, when the calculations for the three test cases are continued to such large distances, it is found that, although they also indicate half-power law behavior, the growth rates are considerably smaller and imply values of $\nu_T/U\theta$ of the order of 0.024. Thus, the k-ϵ model predicts an average eddy viscosity that is approximately 25 to 30% lower than that observed experimentally. Another parameter that is often used to describe the asymptotic growth rate of far wakes (see Rodi[17]) is

$$s = \frac{1}{2}\frac{U}{w_0}\frac{\mathrm{d}b}{\mathrm{d}x} = 2\sqrt{\pi \ln 2}\left(\frac{\nu_T}{U\theta}\right)$$

so that, with $\nu_T/U\theta = 0.032$, the experimental value of s is 0.095, in substantial agreement with the value 0.098 suggested by Rodi after a review of several sets of data. On the other hand, the present calculations yield $s = 0.0645 \pm 0.007$, again about 30% lower.

The disagreement between the calculations and experiments in the far wake is also evident in the velocity and shear-stress profiles. These are shown in Fig. 7 in the form of Eqs. (3) and (4), where the data of Pot at the last measurement station are compared with the asymptotic profiles and those predicted by the present method. It is seen that the predicted profiles do not agree with experiment in the outer half of the wake. Also, the maximum Reynolds stress predicted is about 30% lower than that indicated by the experiment and asymptotic theory.

In summary, the k-ϵ model predicts the development of the near wake, $x/\theta < 350$, say, with satisfactory accuracy but does not lead to the observed asymptotic behavior at large distances. The model fails to correctly describe the velocity and shear-stress profiles in the outer half of the wake. This particular defect of the basic k-ϵ model has also been observed previously, and ad hoc corrections have been suggested as remedies. More specifically, Rodi[6] recommended that the coefficient c_μ in Eq. (12) should be made a function of the average ratio of turbulent energy production to dissipation. Incorporation of this modification in the present calculations led to only a marginal improvement in the profile shapes and increased the growth parameter to $s = 0.068$, still substantially smaller than the experimental value.

A possible explanation for the different performance of the turbulence model in the near and far wakes may lie in the grossly different behavior of the intermittency in the two flow regimes. It is well known that intermittent flow exists in the boundary layer only beyond a distance of 0.4δ from the wall, whereas it penetrates almost up to the centerline of a far wake. The changeover must obviously occur gradually in the near wake. It is not surprising, therefore, to find that a turbulence model constructed essentially for fully turbulent flows and calibrated primarily against such flows continues to predict the boundary-layer-like near wake and breaks down in the highly intermittent far wake.

If the difference in intermittency is indeed the cause of the failure of the turbulence model, then it appears most unlikely that any model based on time-averaged equations would succeed since the necessary information would have already been lost in the averaging process. It may, however, be possible to devise corrections to obtain somewhat improved

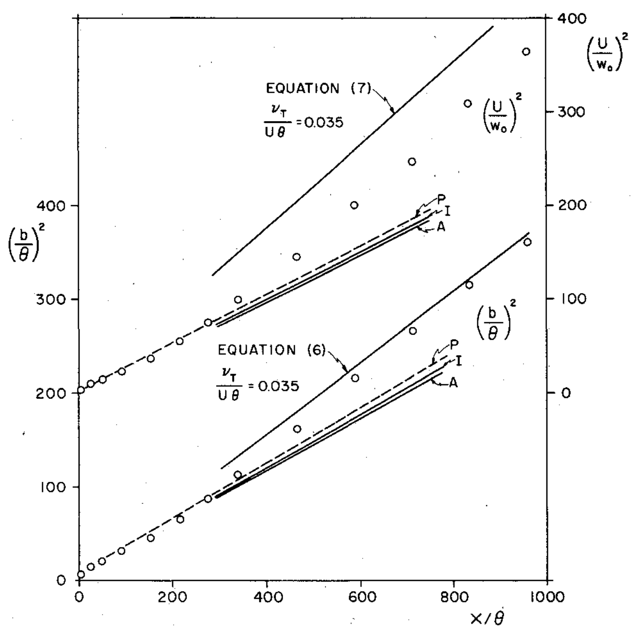

Fig. 6 Asymptotic development; data of Pot.[12] Calculations correspond to Pot (P), Ramaprian et al. (I), and Andreopoulos (A) initial conditions.

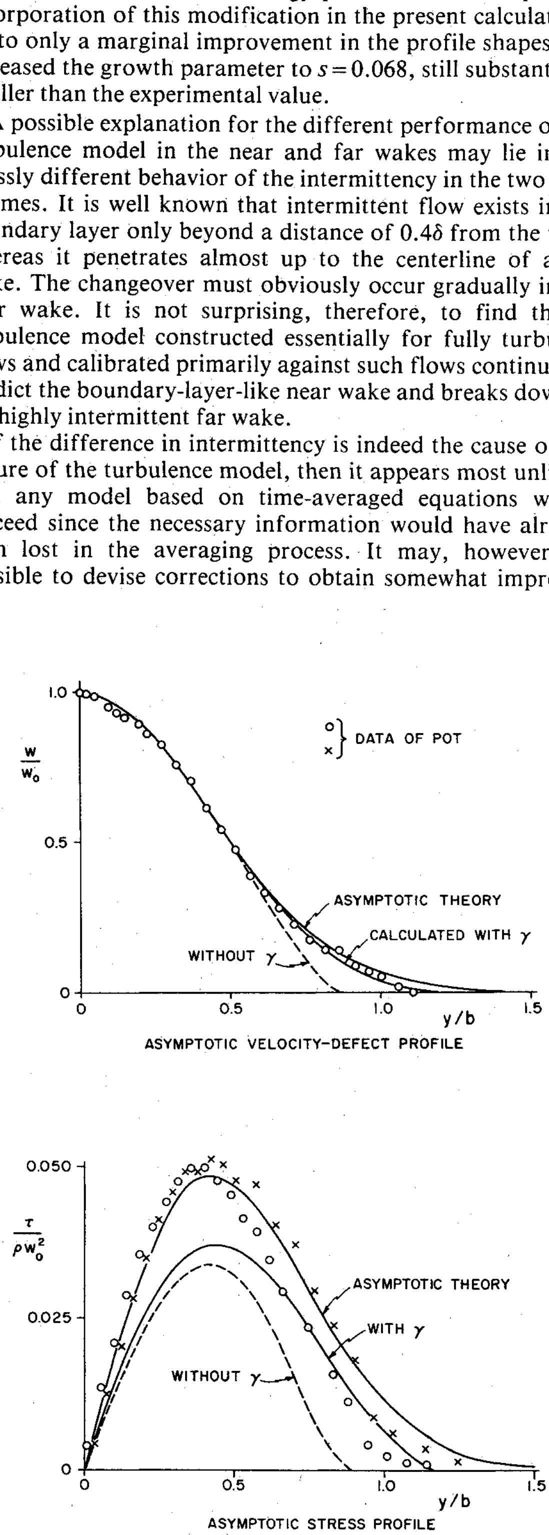

Fig. 7 Asymptotic profiles.

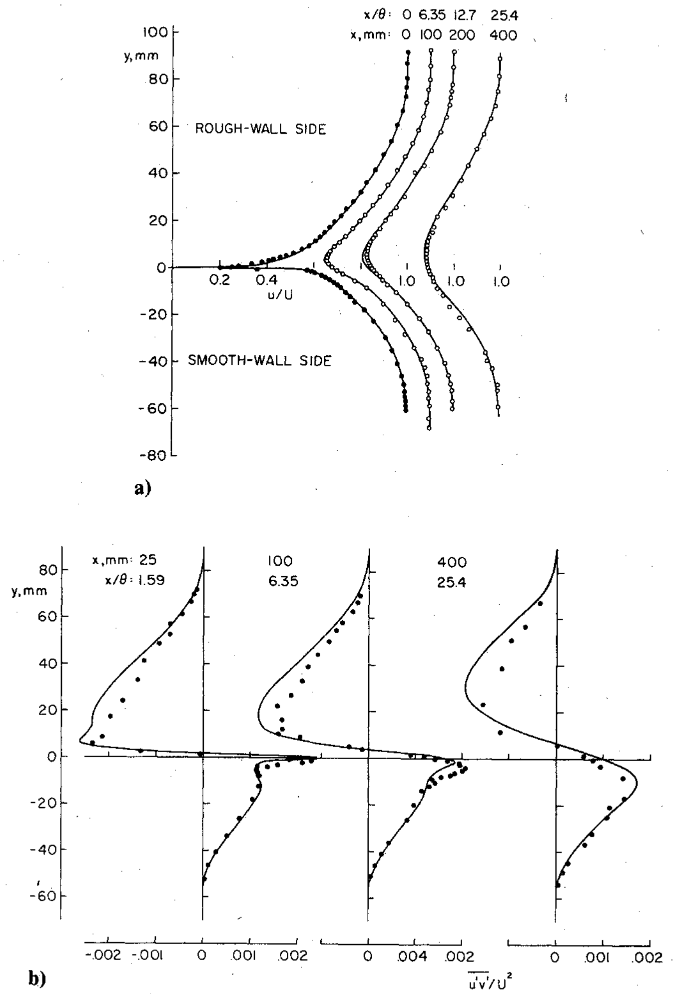

Fig. 8 Asymmetric wake; comparison with data of Andreopoulos.[11] a) Velocity; b) Reynolds shear stress.

predictions. To this end, the present calculations were repeated by using

$$\nu_T = (c_\mu/\gamma)\,(k^2/\epsilon) \tag{12a}$$

and the $\gamma(y/\delta)$ function corresponding to a far wake. The basis for this correction is that Eq. (12) is assumed to apply to fully-turbulent flows and, therefore, in an intermittent flow, k and ϵ should correspond only to the turbulent portion of the motion. This increases k and ϵ by a factor γ^{-1} and hence leads to Eq. (12a). Typical results with this rather simple correction are illustrated in Figs. 4 and 7. It is observed that, as expected, the calculations show poorer agreement in the near wake but a marked improvement in the shapes of the asymptotic velocity and shear-stress profiles (notice also the improvement in far wake H in Fig. 4). Neither the maximum Reynolds stress nor the growth parameters increase to the observed levels, both being about 25% lower than the experimental values (i.e., a 5% improvement). Thus, the proposed correction is not altogether satisfactory although it suggests that a turbulence model that explicitly accounts for the intermittency may be necessary to treat boundary-layer and free shear flows simultaneously.

Asymmetric Wakes

In this case, calculations have been performed for the experimental conditions of Andreopoulos and Ramaprian et al. Considerable care was taken to ensure that the initial conditions in the boundary layers at the trailing edge were

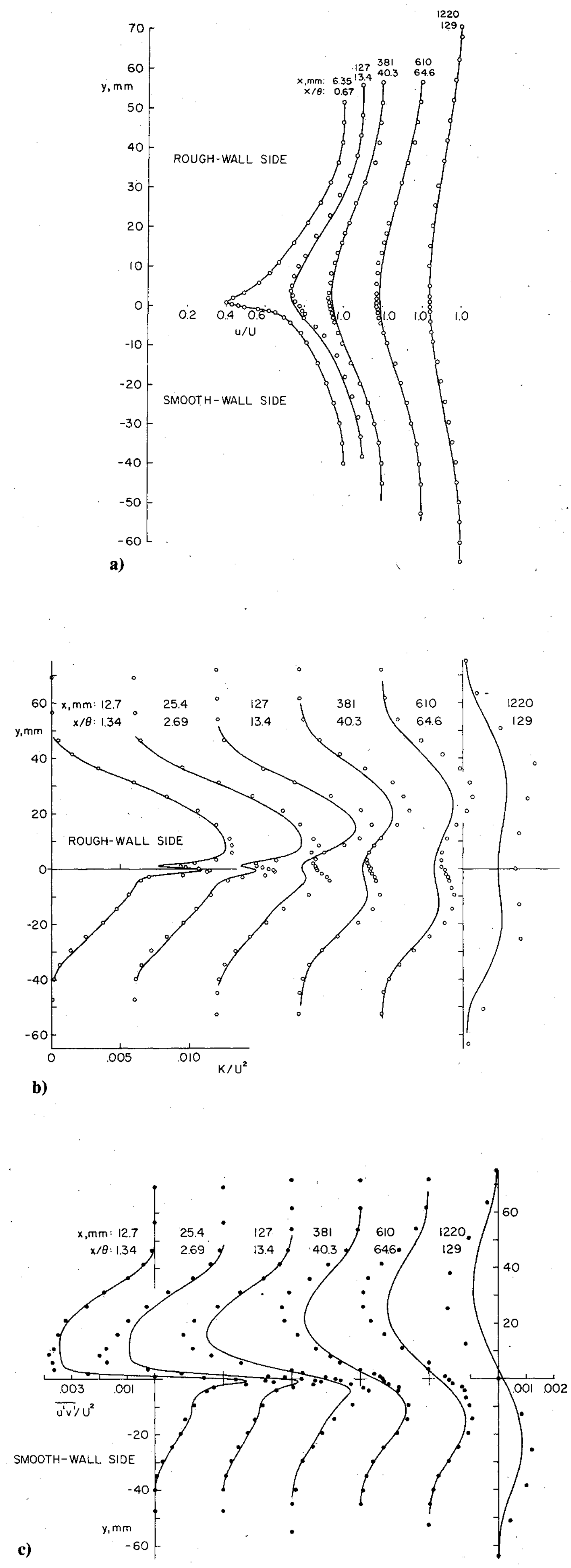

Fig. 9 Asymmetric wake; comparison with data of Ramaprian et al.[13] a) Velocity; b) turbulent kinetic energy; c) Reynolds shear stress.

matched with available data (u, $\overline{u'v'}$ in the case of Andreopoulos, and u, $\overline{u'v'}$, k for Ramaprian et al.). Since both experiments were restricted to near wakes ($x<26$ and 129θ, respectively), the discussion of the approach to asymptotic conditions will be based entirely on the calculations.

Figure 8 shows the velocity and shear-stress profiles at several streamwise positions for the experiments of Andreopoulos. Similar comparisons are made for the measurements of Ramaprian et al. in Fig. 9. The latter also includes the profiles of turbulent kinetic energy since these were also measured. It is seen that the mean-velocity profiles are predicted with fair accuracy in both cases. The corresponding shape parameters, not shown here, are also in excellent agreement with the data.

Perhaps the most interesting feature of these experiments is that both indicate a pronounced peak in the Reynolds stress on the smooth-wall side of the wake and the data of Ramaprian et al. show a corresponding peak in the turbulent kinetic energy. Note that similar peaks were also observed in the symmetric wake experiments of Ramaprian et al. and Pot. The peaks disappear around $x/\theta=25$ in both experiments. This rather curious phenomenon has been attributed by Andreopoulos to the extra production of $\overline{u'v'}$ and k due to the rapid increase in the vertical velocity fluctuations downstream of the trailing edge. As mentioned earlier, however, they could also be associated with local separation and vortex shedding from the trailing edge of finite thickness. Although the issue cannot be resolved without further analysis of the data, it is rather surprising to find that the calculations with the k-ϵ model also predict the peaks in $\overline{u'v'}$ and k, whose magnitudes and locations are in substantial agreement with the experimental data (see Figs. 8b, 9b, and 9c). An examination of the calculated turbulence energy balance shows that the peaks result from rather large local rates of production and diffusion. The agreement between the calculations and experiments in this fine detail is all the more perplexing since the turbulence model does not explicitly contain either of the two physical features mentioned earlier.

The downstream development of the shear-stress profiles in the two experiments, shown in Figs. 8b and 9b, indicates that, up to $x/\theta \sim 25$, the calculations are in general agreement with the data on the smooth-wall side but not on the rough-wall side. On the rough side, the calculated stresses are higher than the data of Andreopoulos but lower than those of Ramaprian et al. Andreopoulos' measurements at $x/\theta=25$, the most downstream station, show a nearly symmetric distribution of stress, whereas his mean-velocity profile, the data of Ramaprian et al., and the calculations all suggest that the asymmetry persists for considerably larger distances. The measurements of Ramaprian et al. at $x/\theta=129$ show that both $\overline{u'v'}$ and k are still asymmetric. The calculations for this case indicate that the predicted stresses and kinetic energy become progressively lower than the measurements, the disagreement being somewhat smaller on the smooth-wall side than on the rough-wall side.

An overview of the results suggests that the present method gives satisfactory predictions in the near wake in most respects but leads to a more rapid adjustment to symmetric conditions than indicated by the data. Also, the lower levels of calculated shear stress and kinetic energy with increasing distance from the trailing edge are similar to the behavior observed in the symmetric wakes. Continuation of the asymmetric wake calculations to much larger distances ($x/\theta \sim 1000$) also leads to essentially the same results with regard to the asymptotic growth rates and velocity and shear-stress distributions as those found in the symmetric case. Thus, both sets of calculations indicate a unique, but incorrect, asymptotic state, independent of the initial conditions at the trailing edge. The more rapid approach of the solutions to this asymptotic state than indicated by the data (Fig. 6) and the disagreement in the asymptotic growth rates and velocity and stress profiles are therefore features of the turbulence model.

Conclusions

The major conclusions from this study may be summarized as follows:

1) Near and far wake flows, together, provide a simple and yet a critical test of the generality of turbulence models.

2) The basic k-ϵ model investigated thus far yields satisfactory agreement with several sets of symmetric and asymmetric near wake data but does not correctly predict either the asymptotic growth rates or the shapes of the velocity, shear-stress, and turbulent kinetic energy profiles in the far wake. Some evidence has been presented to show that this mixed performance of the model may be due to neglect of the marked intermittency of far wakes.

3) The numerical method utilized here extends two-dimensional boundary-layer calculations into symmetric as well as asymmetric wakes without any special difficulty at the trailing edge.

4) The experimentally observed peaks in the turbulence quantities in the immediate vicinity of the trailing edge require further investigation to determine their origin and to incorporate appropriate modifications in the calculations.

Concerning the calculation method, it may be noted that it can be adopted to predict the near wakes of airfoils, which are characterized by pressure gradients and streamline curvature, provided the pressure field is either known or determined iteratively by matching the boundary-layer and wake flows to the external inviscid flow.

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